

On V_{ud} determination from neutron decay

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Abstract

The recent results of the PIBETA experiment strongly suggest the presence of a non- $(V-A)$ anomalous interaction in the radiative pion decay. The same interaction should inevitably contribute to the neutron decay and in particular it should affect the V_{ud} determination. This paper is dedicated to the prediction of the eventual discrepancy in V_{ud} extracted from the super-allowed $0^+ \rightarrow 0^+$ Fermi transitions and from the polarized neutron decay.

1 Introduction

The standard model (SM) includes three generations of quark doublets. However, our world consists of only u and d quarks, the lightest flavours of the first generation. This occurs through quark-generation mixings in charge-changing weak decays. The quark mixings cannot be predicted within the SM and they are matter of experimental investigations and theoretical speculations.

If there exist only three quark generations, then the transition probability, for example, of up -quark u to all $down$ -quarks d , s and b should be equal to one

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1. \quad (1)$$

This unitarity condition can be tested experimentally.

Already in the 1990's [1] it was noted that there is a small deviation from unitarity. Therefore, this issue is attracting great interest and lots of experimental efforts were made to understand this problem. At present experimental accuracy of V_{ud} and V_{us} determination, the third matrix element V_{ub} can be safely neglected in (1), thanks to its small value, and the problem is reduced to a verification of a simple trigonometrical identity

$$\cos^2 \vartheta_C + \sin^2 \vartheta_C = 1 \quad (2)$$

for the Cabibbo angle ϑ_C . Therefore, an independent and self-consistent determination of the first two matrix elements is of great importance.

At present, after a first indication from the E865 Collaboration [2] that the value of V_{us} could be higher than the PDG value 0.2196(23) [3], other experiments [4, 5] also confirm this result. Reanalysis of the hyperon beta decay [6] also leads to a somewhat higher value of $V_{us} \simeq \sin \vartheta_C = 0.2250(27)$, which is in better agreement with the unitarity condition (2). This value surprisingly coincides with the value 0.2238(30) [7] determined from the ratio of experimental kaon and pion decay widths $\Gamma(K \rightarrow \mu\nu) / \Gamma(\pi \rightarrow \mu\nu)$ [3] using the lattice calculations of the pseudoscalar decay constant ratio f_K/f_π [8] and assuming unitarity. All these facts indicate a higher value of V_{us} and hence the unitarity problem does not exist.

Meanwhile, different experiments have been dedicated to independent determinations of the first matrix element V_{ud} . The most precise result, $V_{ud} \simeq \cos \vartheta_C = 0.9740(5)$, comes from a series of experiments on super-allowed $0^+ \rightarrow 0^+$ Fermi transitions [9]. Recently, $V_{ud} = 0.9713(13)$ has been derived, on a comparable precision level, from the polarized neutron decay [10]. A compatible but less precise result $V_{ud} = 0.9728(30)$ [11] has been achieved by measuring the rare pion beta decay.

Although the unitarity problem is certainly solved by now, another problem is probably emerging in connection with a very precise determination of the first matrix element from the super-allowed $0^+ \rightarrow 0^+$ Fermi transitions and from the polarized neutron decay. The present measurements indicate a 2σ difference for the extracted V_{ud} . The situation can be clarified in the future with new experiments for measurements of the angular correlation coefficient a and the asymmetry parameter A in the neutron beta decay at the sub- 10^{-3} level [12].

This paper is dedicated to the prediction of an eventual discrepancy in V_{ud} extracted from the super-allowed $0^+ \rightarrow 0^+$ Fermi transitions and from the polarized neutron decay. The key difference between these two methods is related to the fact that polarization phenomena are very sensitive to chiral structures other than $V - A$. Therefore, a possible new tensor interaction, which explains the anomaly in the radiative pion decay [13], can be responsible for this discrepancy.

2 V_{ud} determination from neutron decay

A determination of the strength of the $u \leftrightarrow d$ quark transition with respect to the pure leptonic $e \leftrightarrow \nu_e$ transition can be made using absolute measurements, *e.g.* partial widths or (and) lifetime. Since the neutron has only one decay mode, $n \rightarrow p e \bar{\nu}(\gamma)$, precise measurements of its mean lifetime are absolutely necessary. Nevertheless, the knowledge of only this parameter cannot allow us to determine V_{ud} , because in the SM the neutron lifetime τ_n ,

$$\tau_n^{-1} \propto |V_{ud}|^2 G_F^2 (1 + 3|\lambda|^2), \quad (3)$$

depends also on the phenomenological parameter λ , the ratio of the axial coupling constant to the vector coupling constant.¹

Fortunately, in the neutron decay several observables are accessible to experiments, which also depend on the same parameter λ . For example, the decay probability for a polarized neutron can be written [15] as

$$\frac{d\Gamma}{dE_e d\Omega_e d\Omega_{\bar{\nu}}} \propto 1 + a \frac{\mathbf{p}_e \cdot \mathbf{p}_{\bar{\nu}}}{E_e E_{\bar{\nu}}} + \langle \boldsymbol{\sigma}_n \rangle \cdot \left[A \frac{\mathbf{p}_e}{E_e} + B \frac{\mathbf{p}_{\bar{\nu}}}{E_{\bar{\nu}}} + D \frac{\mathbf{p}_e \times \mathbf{p}_{\bar{\nu}}}{E_e E_{\bar{\nu}}} \right], \quad (4)$$

where $\langle \boldsymbol{\sigma}_n \rangle$ is the neutron polarization. The correlation coefficients a , A , B and D in the SM are given by the relations:

$$a = \frac{1 - |\lambda|^2}{1 + 3|\lambda|^2}, \quad A = -2 \frac{|\lambda|^2 + \text{Re}(\lambda)}{1 + 3|\lambda|^2}, \quad B = 2 \frac{|\lambda|^2 - \text{Re}(\lambda)}{1 + 3|\lambda|^2}, \quad D = 2 \frac{\text{Im}(\lambda)}{1 + 3|\lambda|^2}. \quad (5)$$

A non-zero value of the triple correlation coefficient D would indicate T -violation; however, in the SM its value is predicted to be vanishingly small. Present experiments confirm this statement at the level of about 0.1%. Therefore, in the following we will consider the parameter λ to be a real constant.

Taking into account the current world average value for $\lambda = -1.2695(29)$ [3] we can estimate the sensitivity of the correlation coefficients to this parameter

$$\frac{\delta a}{a} = \frac{8\lambda^2}{(\lambda^2 - 1)(1 + 3\lambda^2)} \frac{\delta\lambda}{\lambda} \simeq 3.6 \frac{\delta\lambda}{\lambda}, \quad (6)$$

$$\frac{\delta A}{A} = \frac{(1 - \lambda)(1 + 3\lambda)}{(1 + \lambda)(1 + 3\lambda^2)} \frac{\delta\lambda}{\lambda} \simeq 4.1 \frac{\delta\lambda}{\lambda}, \quad (7)$$

$$\frac{\delta B}{B} = \frac{(1 + \lambda)(1 - 3\lambda)}{(1 - \lambda)(1 + 3\lambda^2)} \frac{\delta\lambda}{\lambda} \simeq -0.1 \frac{\delta\lambda}{\lambda}. \quad (8)$$

The parameters most sensitive to λ appear to be the angular correlation coefficient a and the asymmetry parameter A . Therefore, in the following we will concentrate only on these correlation coefficients.

The recent results of the PIBETA experiment on radiative pion decay [16] strongly suggest the presence of non- $(V - A)$ anomalous interaction [13]

$$\mathcal{L}_T = -f_T \frac{G_F V_{ud}}{\sqrt{2}} \bar{u} \sigma_{\lambda\alpha} d \frac{4q_\alpha q_\beta}{q^2} \bar{e} \sigma_{\lambda\beta} (1 - \gamma^5) \nu_e + \text{h.c.}, \quad (9)$$

¹For the sake of simplicity we neglect, in the following, all effects of the radiative corrections and additional terms of recoil order, *e.g.* weak magnetism. However, such terms should be taken into account in precision measurements of neutron decay [14].

with dimensionless coupling constant $f_T \simeq 0.013$, where q_α is the momentum transfer between quarks and leptons. It is obvious that the same interaction will affect all observables of the neutron decay: the lifetime (3) and the decay distribution (4).

In order to apply this interaction to the neutron decay we estimate the matrix element

$$\langle p | \bar{u} \sigma_{\alpha\beta} d | n \rangle = g_T \bar{p} \sigma_{\alpha\beta} n \quad (10)$$

using the technique of ref. [17] with $g_T \simeq 4.7(m_u + m_d)/(m_n + m_p) = 0.029(8)$ for the current quark masses $m_u + m_d = 11.5 \pm 3.4$ MeV at 1 GeV [3]. This leads to a new matrix element in the neutron decay

$$\mathcal{M}_T = -F_T \frac{G_F V_{ud}}{\sqrt{2}} \bar{p} \sigma_{\lambda\alpha} n \frac{q_\alpha q_\beta}{q^2} \bar{e} \sigma_{\lambda\beta} (1 - \gamma^5) \nu_e, \quad (11)$$

with an effective coupling constant $F_T = 4f_T g_T = 1.5(4) \times 10^{-3}$, which is an order of the recoil effects $(m_n - m_p)/m_n \equiv \Delta/m_n \approx 1.4 \times 10^{-3}$.

It is worth while to note that the matrix element (11) consists of two different terms with opposite nucleon chiralities

$$\mathcal{M}_T = -F_T \frac{G_F V_{ud}}{4\sqrt{2}} \bar{p}_R \sigma_{\lambda\beta} n_L \bar{e} \sigma_{\lambda\beta} (1 - \gamma^5) \nu_e \quad (12)$$

$$- F_T \frac{G_F V_{ud}}{\sqrt{2}} \bar{p}_L \sigma_{\lambda\alpha} n_R \frac{q_\alpha q_\beta}{q^2} \bar{e} \sigma_{\lambda\beta} (1 - \gamma^5) \nu_e. \quad (13)$$

The first term (12) is the usual local tensor matrix element, which has been used in the literature for testing a possible effect of new interactions. The second term (13) is a new non-local tensor matrix element originated from the non-local quark interaction (9), which has been constructed so as to avoid the constraints from the ordinary pion decay [18].

3 Unpolarized neutron decay

Taking into account the new matrix element (11), the differential energy distribution of an unpolarized neutron decay at rest reads

$$\begin{aligned} \frac{d^2 \Gamma_0}{dE_e dq^2} &\propto 2E_e (E_m - E_e) (1 + \lambda^2) - \frac{q^2 - m_e^2}{2} (1 - \lambda^2) \\ &+ 2F_T \left[2E_e (E_m - E_e) \frac{m_e}{E_e} - \frac{q^2 - m_e^2}{q^2} m_e E_m \right] \lambda \\ &+ F_T^2 \left\{ 2E_e (E_m - E_e) - \frac{q^2 + m_e^2}{4} - \left[E_m (E_m - 2E_e) - \frac{m_e^2}{2} \right] \frac{m_e^2}{q^2} - E_m^2 \frac{m_e^4}{q^4} \right\}, \end{aligned} \quad (14)$$

where $E_m = (m_n^2 - m_p^2 + m_e^2)/(2m_n) \approx \Delta$ is the maximum electron energy, and the squared momentum transfer $q^2 = \Delta^2 - 2m_n T$ is connected to the proton kinetic energy T or to the electron-antineutrino angular correlation $q^2 = m_e^2 + 2(E_e E_{\bar{\nu}} - \mathbf{p}_e \cdot \mathbf{p}_{\bar{\nu}})$.

The first line in the r.h.s. of eq. (14) is the SM contribution in the limit of the non-relativistic nucleons. The second line presents the contribution from an interference between the SM matrix element and the new tensor matrix element (11). As expected this

contribution is proportional to λ , because the tensor interaction interferes only with the Gamow–Teller amplitude. It consists of two terms: the well-known Fierz interference term and the new contribution from the tensor interaction with opposite nucleon chiralities (13). The third line stems from the square of the new matrix element (11) and can be safely neglected thanks to the second-order contribution of the small parameter F_T .

On the one hand angular correlation measurements have the advantage that it is not necessary to deal with a polarized neutron. On the other hand, since the antineutrino is not registered, only indirect methods have been employed through a detection of low-energy recoil protons. The present accuracy of a does not exceed 5%, which corresponds to a worse λ uncertainty $\Delta\lambda \simeq 0.0139$ than that extracted from the asymmetry parameter A [3].

Meanwhile, new experiments have been proposed [19, 20] with an improved accuracy of a -measurements, using different experimental methods. The aim of the collaboration [19] is to use the neutron decay spectrometer *a*SPECT to improve the precision of a by more than an order of magnitude, relying on the method based on measurements of the proton kinetic energy spectrum. However, even at this precision, it will be impossible to detect the contribution of the new matrix element (11).

It is interesting to note that an integration over the electron energy spectrum leads to a zero result for the interference term in the second line in eq. (14), because the two different contributions cancel each other. Therefore, the new tensor interaction (11) does not distort the recoil proton spectrum and does not contribute to the neutron lifetime. The results of the experiment should correspond to the SM predictions.

A different situation may occur in the case of direct measurements of a by recording the spectrum of electrons emitted into a given range of angles referred to the proton momentum [20]. However, the expected 1% accuracy in the value of a will probably be insufficient to detect the effect of the new tensor interaction (11) due to small coupling constant F_T .

4 Decay of the polarized neutron

The decay of the polarized neutron allows us to extract λ only from the well-measured electron spectrum

$$\frac{d\Gamma}{dE_e} = \frac{d\Gamma_0}{dE_e} + \langle \sigma_n \rangle \cdot \frac{\mathbf{p}_e}{E_e} \frac{d\Gamma_A}{dE_e}, \quad (15)$$

where

$$\begin{aligned} \frac{d\Gamma_0}{dE_e} &\propto E_e (E_m - E_e) (1 + 3\lambda^2) \\ &+ 2m_e F_T \left[E_m - 2E_e + E_m \frac{m_e^2}{2b} \ln \left| \frac{a+b}{a-b} \right| \right] \lambda \end{aligned} \quad (16)$$

can be obtained from eq. (14), neglecting the last term and integrating over q^2 . Here the functions $a = 2E_e(E_m - E_e) + m_e^2$ and $b = 2(E_m - E_e)|\mathbf{p}_e|$ are related to the maximum $q_{\max}^2 = a + b$ and the minimum $q_{\min}^2 = a - b$ of the squared momentum transfer.

The anisotropic distribution in the presence of the new tensor interaction (11) can be easily calculated:

$$\begin{aligned}
\frac{d\Gamma_A}{dE_e} \propto & -2E_e(E_m - E_e)(\lambda^2 + \lambda) \\
& - \frac{m_e F_T E_e}{E_e^2 - m_e^2} \left\{ E_e(E_m - E_e) + \frac{m_e^2}{2} - \left[E_m(E_m - E_e) + \frac{m_e^2}{4} \right] \frac{m_e^2}{b} \ln \left| \frac{a+b}{a-b} \right| \right. \\
& \left. - \left[E_e(E_m + E_e) - \frac{m_e^2}{2} - \left(E_m^2 - \frac{m_e^2}{4} \right) \frac{m_e^2}{b} \ln \left| \frac{a+b}{a-b} \right| \right] \lambda \right\}, \quad (17)
\end{aligned}$$

where only the SM contribution and the leading-order term in F_T are shown as well. This new term stems only from the interference between the SM matrix element and the tensor interaction for the transition of the right-handed neutron to the left-handed proton (13). It is absent in the usual case of the local tensor matrix element for the transition of the left-handed neutron to the right-handed proton (12).

The new contribution in eq. (17) is negative over the whole electron spectrum and it leads effectively to a larger absolute value of the asymmetry parameter A than in the SM for the same parameter λ . Therefore, to extract the right value of λ , the experimental asymmetry should be fitted according to eqs. (15), (16), (17), taking into account the contribution of the new tensor interaction.

To estimate the effect of this interaction we plot in fig. 1 the ratio of the asymmetry parameter $A = d\Gamma_A/d\Gamma_0$ to its value A_0 at $F_T = 0$ for the region of the electron spectrum fitted by the PERKEO II Collaboration [10]. It shows in average 0.7% systematic contribution from the tensor interaction, which is the same as the accuracy of the experiment. Therefore, the real value of the measured parameter $\lambda = 1.2720(19)$ can be obtained from the experimental value $\lambda^{\text{exp}} = 1.2739(19)$ by shifting it down with the value of the experimental accuracy, $\Delta\lambda = 0.0019$.

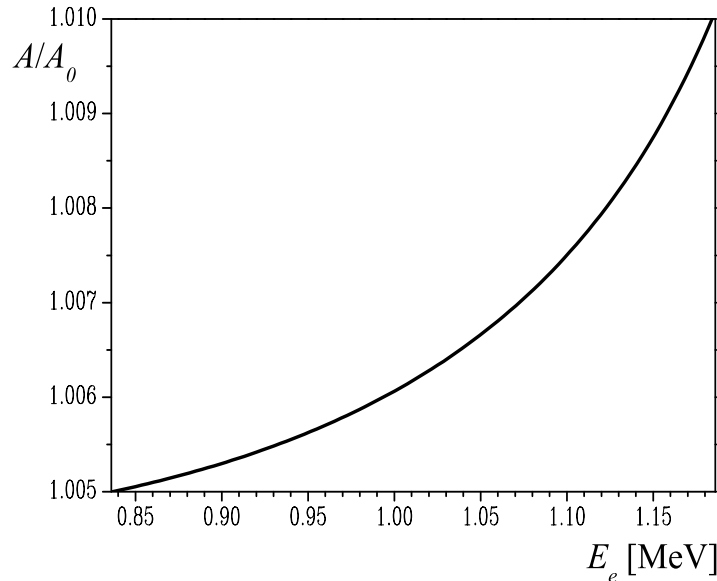


Figure 1: The ratio of the asymmetry parameter $A = d\Gamma_A/d\Gamma_0$ to its value A_0 at $F_T = 0$ for the region of the electron spectrum fitted by the PERKEO II Collaboration.

Taking into account all effects from the radiative corrections and the phase-space factor, and also the fact that the new tensor interaction does not affect, in the leading order, the neutron lifetime, eq. (3) can be rewritten as [21]

$$|V_{ud}|^2 = \frac{4908 \pm 4s}{\tau_n (1 + 3\lambda^2)}. \quad (18)$$

Using $\tau_n = 885.7(8)s$ and $\lambda = 1.2720(19)$, it leads to the corrected value $V_{ud} = 0.9729(13)$ extracted from the polarized neutron decay, which is in a better agreement with $V_{ud} = 0.9740(5)$ extracted from super-allowed beta decays.

5 Conclusions

Recently strong evidence for a deviation from the SM has been obtained by the PIBETA Collaboration [16]. Namely, the SM fails to describe the energy distribution and the branching ratio of the radiative decays of positive pions at rest in the high- E_γ /low- E_e kinematic region of the Dalitz plot. The previous experiment, performed by the ISTRAC Collaboration [22], testing the radiative decays of negative pions in flight in a wide kinematic region, had announced the same effect, although statistically less significant. The present PIBETA result [16] indicates a deficit of the branching ratio of the radiative pion decay in the specified kinematic region at the 8σ level with respect to the SM prediction, while in the other kinematic regions both the branching ratios and the energy distributions are compatible with the $V-A$ interaction.

The anomaly observed by the ISTRAC Collaboration has been explained in the framework of an extended theory of the electroweak interactions, with a new type of fundamental particles – chiral spin-1 bosons [18] described by antisymmetric second-rank tensor fields. An exchange of these particles leads effectively to the phenomenological tensor interaction (9). The same interaction should contribute to the neutron decay as well. However, it has been shown here that with the present experimental accuracy we cannot conclude definitely about their presence in the neutron decay. The only hint of their manifestation is the partial explanation of the about 2σ discrepancy in V_{ud} extracted from the polarized neutron decay and from the super-allowed beta decays. Probably, new experiments under construction, *e.g.* the UNCA and the abBA, using new Spallation Neutron Source facilities and aiming at high-precision measurements of the neutron decay parameters, could definitely confirm or reject the predicted distortions of the spectrum due to the new tensor interaction.

Based on the universality of couplings of the new chiral bosons to leptons and quarks, the extended electroweak model [18] predicts an admixture of analogous tensor interactions in pure lepton processes [23]. However, at present it is not easy to detect them in the ordinary muon decay [24] as well. Meanwhile, the new tensor interaction (9), with the same effective coupling constant, allows us to explain [25] the painful widely commented discrepancy in the two-pion spectral functions extracted from the e^+e^- annihilation and from the τ -decay. It is hoped that future experiments will clarify this situation better.

Acknowledgements

The author acknowledges the warm hospitality of the Theory Division at CERN, where this work has been fulfilled.

References

- [1] W. Jaus and G. Rasche, *Phys. Rev. D* **41** (1990) 166.
- [2] A. Sher *et al.*, *Phys. Rev. Lett.* **91** (2003) 261802.
- [3] S. Eidelman *et al.*, *Phys. Lett. B* **592** (2004) 1.
- [4] S. Miscetti (KLOE Collaboration), hep-ex/0405040.
- [5] T. Alexopoulos *et al.* (KTeV Collaboration), *Phys. Rev. Lett.* **93** (2004) 181802.
- [6] N. Cabibbo, E. C. Swallow and R. Winston, *Annu. Rev. Nucl. Part. Sci.* **53** (2003) 39; hep-ph/0307214.
- [7] W. J. Marciano, hep-ph/0402299.
- [8] C. Aubin *et al.* (MILC Collaboration), hep-lat/0309088; hep-lat/0310041.
- [9] I. S. Towner and J. C. Hardy, *J. Phys. G* **29** (2003) 197.
- [10] H. Abele *et al.*, *Phys. Rev. Lett.* **88** (2002) 211801.
- [11] D. Počanić *et al.* (PIBETA Collaboration), *Phys. Rev. Lett.* **93** (2004) 181803.
- [12] H. Abele and D. Mund (eds.), Proceedings of Workshop on *Quark-Mixing, CKM-Unitarity*, Heidelberg, 2002 (Mattes-Verlag, Heidelberg, 2003).
- [13] M. V. Chizhov, hep-ph/0402105.
- [14] S. Gardner, in [12], p. 113.
- [15] J. D. Jackson, S. B. Treiman and H. W. Wyld, Jr., *Phys. Rev.* **106** (1957) 517.
- [16] E. Frlež *et al.* (PIBETA Collaboration), *Phys. Rev. Lett.* **93** (2004) 181804.
- [17] A. A. Poblaguev, *Phys. Lett. B* **238** (1990) 108.
- [18] M. V. Chizhov, *Mod. Phys. Lett. A* **8** (1993) 2753.
- [19] S. Baeßler *et al.*, in [12], p. 63.
- [20] B. G. Yerozolimsky, in [12], p. 57.
- [21] A. Czarnecki, W. J. Marciano and A. Sirlin, hep-ph/0406324.
- [22] V. N. Bolotov *et al.*, *Phys. Lett. B* **243** (1990) 308.
- [23] M. V. Chizhov, hep-ph/0405073.
- [24] J. R. Musser *et al.* (TWIST Collaboration), hep-ex/0409063;
A. Gaponenko *et al.* (TWIST Collaboration), hep-ex/0410045.
- [25] M. V. Chizhov, hep-ph/0311360.